Analysis of oscillating compressible flow through a packed bed

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The forced convective flow of an ideal gas through a packed bed was investigated by finite-difference numerical means. Oscillating gas-phase temperature/pressure inlet boundary conditions were considered. The effect of oscillating boundary conditions on the transport phenomena in the packed bed was investigated and comparisons were made with the case of constant-temperature and constant-pressure boundary conditions. The average energy storage characteristics were found to be very close in both oscillating and constant inlet boundary conditions.

Keywords: packed bed; compressible; oscillating inlet conditions; energy storage

Introduction

In storage of thermal energy in packed beds, the dynamic behavior of the packed-bed system is an important consideration. Ideal conditions of constant-temperature and constant-pressure inlet conditions are quite difficult to maintain. Therefore, examining those conditions that more closely approximate the real-life situations, such as variable-pressure or variable-temperature inlet conditions, provides better insight to such problems.

Packed beds have been widely used in engineering for heat and mass transfer applications and for energy storage purposes. The principles that are used to study transport phenomena in packed beds are the same as those used for porous media in general. The related applications are very common in chemical engineering processes as well as petroleum, geothermal, and nuclear engineering processes. Related mechanical engineering applications mostly deal with thermal energy storage systems.

Extensive literature is available on the applications of packed beds and porous media. The majority of these investigations deal with incompressible flows through packed beds or other forms of porous materials. The simplest model used for analysis of transport phenomena in porous media is the so-called one-phase model, in which the porous medium and the working fluid are approximated as a quasi-homogeneous medium with properly defined effective transport coefficients. The basis of the two-phase model, in which the assumption of local thermal equilibrium between the fluid and solid phases is not used, is the Schumann[1] model. Riaz[2] reported a comparison between the single- and two-phase model solutions. Because of the incompressible flow assumption and neglect of boundary effects, these models naturally involve only energy equations. A number of simplifications such as neglecting axial conduction terms have been employed in most of the investigations dealing with two-phase models. Spiga and Spiga[3] reported an investigation in which different boundary conditions were considered.

The majority of the investigations dealing with compressible flow through packed beds or other porous media concentrate on ideal-gas behavior, and employ the assumption of local thermal equilibrium between the solid and fluid phases. Analytical solutions for simplified cases have been reported by Kidder[4] for isothermal flow of an ideal gas through a porous medium, and by Morrison[5] for isothermal and adiabatic flows of an ideal gas through a porous medium. These studies do not take into account the inertia effects, and they utilize the local thermal equilibrium (LTE) assumption, i.e., the one-phase model. Another study that incorporates the LTE assumption but accounts for the inertia effects was reported by Nilson.[6]

In establishing the condition of no local thermal equilibrium between the solid and fluid phases, energy equations are developed by the use of a heat transfer term in each, representing the interphase heat transfer formulated by the use of the fluid-to-particle heat transfer coefficient. Extensive studies have been conducted for establishing empirical correlations for the fluid-to-particle heat transfer coefficients for packed beds of different geometry, packing configuration, and particle size, and related literature surveys are also available.[7,8]

Rigorous models have been developed by Vafai and Sözen[9] and Sözen and Vafai[10] for the forced convective flow of a superheated ideal gas and forced convective condensing flow of a vapor through a packed bed. The former of these studies concentrated on the parameters influencing the LTE and two-dimensionality of the transport phenomena, while the latter concentrates on the condensing flows through a packed bed. The LTE assumption has not been used in any of these studies, and inertia effects have been accounted for by the use of the Ergun–Forchheimer relation rather than Darcy flow formulation.

In the present study, compressible flow of an ideal gas through a packed bed is investigated for oscillating inlet-pressure and oscillating inlet-temperature boundary conditions in accordance with the fact stated earlier that such boundary conditions represent more closely the real-life conditions for certain situations. For example, in real applications, more often than not, some form of oscillation prevails in the inlet pressure or temperature. It is then crucial to know the qualitative and quantitative effects of the oscillations on the thermal charging characteristics and the net energy storage capabilities of the packed bed. Our aim is to analyze the behavior of the transport processes and energy storage characteristics in these oscillating boundary condition flows through porous media with specific attention on packed beds.

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Received 6 August 1990; accepted 9 November 1990

Problem statement and formulation

A rectangular packed bed is assumed to be formed by regularly shaped and sized spheres packed between two horizontal walls. The schematic diagram of the physical system under consideration is depicted in Figure 1. Relatively high-speed flows are considered in the present study and, therefore, the flow is essentially forced convective in nature. In the present study, the top and bottom walls are assumed to be insulated and the depth to be infinitely long, thus rendering the problem essentially one-dimensional (1-D). The working fluid was taken to be superheated Refrigerant-12, which was modeled as an ideal gas, while the material of the packed-bed particles was chosen to be 1% carbon-steel. The governing equations following ref. 9 are given as follows:

$$\frac{\partial}{\partial t}(\varepsilon\langle\rho_v\rangle^v)+\frac{\partial}{\partial x}(\langle\rho_v\rangle^v\langle u_v\rangle)=0 \tag{1}$$

$$\frac{\partial\langle P_v\rangle^v}{\partial x}=-\frac{\langle\rho_v\rangle^v F\varepsilon}{K_v^{1/2}}\langle u_v\rangle^2-\frac{\mu_v}{K_v}\langle u_v\rangle \tag{2}$$

$$\varepsilon\langle\rho_v\rangle^v c_{p_v}\frac{\partial\langle T_v\rangle^v}{\partial t}+c_{p_v}\langle\rho_v\rangle^v\langle u_v\rangle\frac{\partial\langle T_v\rangle^v}{\partial x}=\frac{\partial}{\partial x}\left\{k_{\text{veff}}\frac{\partial\langle T_v\rangle^v}{\partial x}\right\}+h_{sv}a_{sv}\{\langle T_s\rangle^s-\langle T_v\rangle^v\} \tag{3}$$

$$(1-\varepsilon)\rho_s c_{p_s}\frac{\partial\langle T_s\rangle^s}{\partial t}=\frac{\partial}{\partial x}\left\{k_{\text{seff}}\frac{\partial\langle T_s\rangle^s}{\partial x}\right\}-h_{sv}a_{sv}\{\langle T_s\rangle^s-\langle T_v\rangle^v\} \tag{4}$$

$$\langle P_v\rangle^v=\langle\rho_v\rangle^v R\langle T_v\rangle^v \tag{5}$$

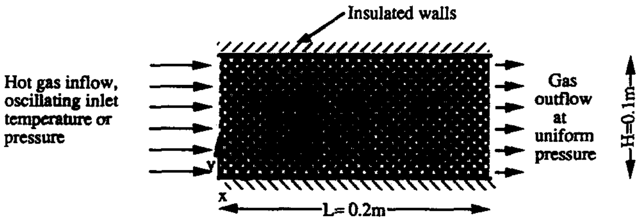

Figure 1 Schematic diagram of the problem

These equations represent the gas-phase continuity equation, gas-phase momentum equation, gas-phase energy equation, solid-phase energy equation, and the equation of state for the working fluid, respectively, where the unknown variables solved from these equations are, respectively, $\langle\rho_v\rangle^v$, $\langle u_v\rangle$, $\langle T_v\rangle^v$, $\langle T_s\rangle^s$, and $\langle P_v\rangle^v$.

Modeling of several geometric parameters and effective thermophysical properties have been based on previous investigations. The permeability of the packed bed, K_v, and the geometric function F appearing in the gas-phase momentum equation were modeled as follows, based on the works of Ergun[11] and Vafai:[12]

$$K_v=\frac{\varepsilon^3 d_p^2}{150(1-\varepsilon)^2} \tag{6}$$

$$F=\frac{1.75}{\sqrt{150}\varepsilon^{3/2}} \tag{7}$$

The surface area of the particles per unit volume of the packed bed was formulated as[13]

$$a_{sv}=\frac{6(1-\varepsilon)}{d_p} \tag{8}$$

The particle diameter was taken to be 2 mm. Following the experimental findings of Benanati and Brosilow,[14] the average value of the porosity of the packed bed was taken to be 0.39.

Modeling of the problem was completed by formulating the effective thermal conductivities and the fluid-to-particle heat transfer coefficient. The former ones were established as

$$k_{\text{veff}}=\varepsilon k_v \qquad k_{\text{seff}}=(1-\varepsilon)k_s \tag{9}$$

and the latter one was obtained from the work of Gamson et al.[15] in the following form:

$$h_{sv}=1.064c_pG\left(\frac{c_p\mu}{k}\right)^{-2/3}\left(\frac{d_pG}{\mu}\right)^{-0.41} \quad \text{for } \frac{d_pG}{\mu}>350 \tag{10}$$

Considering the size of the particles of the packed bed and the range of particle Reynolds numbers employed in the present investigation, the above correlation was found to be the most

Notation

- A Amplitude of the pressure variation, kPa
- a_{sv} Specific surface area of the bed particles, m^2/m^3
- B Amplitude of the temperature variation, K
- c_p Specific heat at constant pressure, J/kg·K
- d_p Particle diameter, m
- f Frequency of oscillation, Hz
- F Geometric factor defined in Equation 2
- G ρu, mass flux, kg/m^2·s
- h_{sv} Fluid-to-particle heat transfer coefficient, W/m^2·K
- H Height of the packed bed, m
- k Thermal conductivity, W/m·K
- K Permeability, m^2
- L Length of the packed bed, m
- P Pressure, N/m^2
- R Gas constant for Refrigerant-12, J/kg·K
- Re_p Particle Reynolds number
- t Time, s
- T Temperature, K
- u Velocity component in x-direction, m/s

Greek letters

- ε Porosity
- μ Absolute viscosity, kg/m·s
- ρ Density, kg/m^3

Subscripts

- av Average inlet
- o Initial
- s Solid
- v Gas
- seff Effective property for solid
- veff Effective property for vapor

Superscripts

- s Solid
- v Gas

Symbols

- $\langle\ \rangle$ "Local volume average" of a quantity appropriate one to use. Moreover, it compared reasonably well with the most recent correlations for the fluid-to-particle heat transfer coefficient.

The initial conditions employed in the solution of the problem were

$$P_v(x, t=0)=P_0$$
$$T_v(x, t=0)=T_s(x, t=0)=T_0 \tag{11}$$
$$u_v(x, t=0)=0$$

with corresponding values of ρ_v being computed from Equation 5.

The boundary conditions used for the case with oscillating inlet gas pressure can be mathematically expressed as

$$P_v(x=0, t)=P_{av}+A\cos(2\pi ft)$$
$$P_v(x=L, t)=P_0 \tag{12}$$
$$T_v(x=0, t)=T_{vav}$$

where f is the frequency and A is the amplitude of the cosinusoidal variation of the inlet pressure, and again the corresponding ρ_v values are computed from Equation 5.

The boundary conditions employed for the case with the oscillating gas-phase inlet temperature condition can similarly be expressed as

$$P_v(x=0, t)=P_{av}$$
$$P_v(x=L, t)=P_0 \tag{13}$$
$$T_v(x=0, t)=T_{vav}+B\cos(2\pi ft)$$

where f is the frequency and B is the amplitude of the variation of the inlet gas temperature. ρ_v is computed as in previous cases. The boundary conditions for the case with constant temperature and constant pressure at the inlet are similar to those in Equation 12 with A being equal to zero.

In the case studies performed, the following values have been used for the variables:

$P_0=100$ kPa

$P_{av}=104$ kPa

$T_0=300$ K

$T_{vav}=350$ K

The nominal particle Reynolds number for the average inlet pressure and temperature was 745. The other thermophysical properties used in the case studies were as follows:

Refrigerant-12
$c_p=710$ J/kg·K

$k=0.0097$ W/m·K

$\mu=12.6\times10^{-6}$ kg/m·s

$R=68.7588$ J/kg·K

1% Carbon-steel
$c_p=473$ J/kg·K

$k=43$ W/m·K

$\rho=7800$ kg/m^3

Solution method

Equations 1 to 5 were solved by using an explicit-scheme finite-difference method. The temporal derivative terms were approximated by forward Euler differencing while most of the spatial derivative terms were approximated by central differencing for all the inner grid points. An exception to the latter approximation was the use of forward differencing in approximating the pressure gradient term in the gas-phase momentum equation. This was done to ensure the stability of the numerical solution for the *early stage* of the problem. Another exception was the use of first-order upwind differencing in the convective terms of the gas-phase continuity and energy equations. However, central differencing could also be used in the convective term of the gas-phase continuity equation. Forward and backward differencing were employed for the spatial derivative terms for the left and right boundary grid points, respectively.

The common procedure of ensuring the stability and accuracy of the explicit schemes by choosing a proper combination of Δx and Δt was utilized in this work. This was done by systematically decreasing the grid size and selecting Δt in such a way to ensure convergent solution. The grid size was decreased until the difference between the solution obtained by the chosen grid size and that obtained by decreasing the grid size by half did not exceed 1%. A 41×1 grid configuration was found to satisfy this criterion. Therefore, 41 grid points were used in the x-direction.

A similar numerical code has been used by the authors in their previous investigations, and the analytical solutions for two simplified cases of transport phenomena in porous media as given by refs. 2 and 4 were compared against the solutions obtained by their code.[9] Ref. 2 presented an analytical solution for an incompressible flow through a porous medium, while ref. 4 presented an analytical solution for isothermal flow of a gas through a porous medium. The agreement between the numerical solutions and the analytical ones was excellent.

Results and discussions

In order to explore any differences in the transport phenomena and energy storage characteristics of the packed bed with oscillating-flow boundary conditions from those of constant boundary conditions, we considered these cases for different ranges by using different values for parameters A, f, and B in Equations 12 and 13 in different runs.

In their previous investigations the authors have observed that for constant-temperature and constant-pressure boundary conditions at the inlet, the solution of the problem had two distinct stages[9,10]—namely, the *early stage* during which the pressure evolution in the packed bed takes place very rapidly, causing drastic variation in the other field variables, and the *later stage* in which the change in the field variables are mostly temperature dependent. The distribution of different field variables along the packed bed during the early stage are depicted in Figure 2 for the case with constant inlet temperature and pressure. In this figure the gas-phase density was nondimensionalized with respect to an average density value calculated from the equation of state using average values of gas-phase pressure and temperature, i.e., average of the initial and inlet conditions. Likewise, the nominal value of the velocity used for nondimensionalizing the gas-phase velocity was computed from Equation 2 based on the average density and the average pressure gradient along the packed bed. These characteristics have been discussed in detail in refs. 9 and 10. Similar characteristics were observed in the present work, with the exception that in the case of the oscillating inlet-pressure condition, the pressure distribution within the packed bed was not very closely linear as it was in the other cases. Rather, it showed an oscillating behavior along the packed bed.

First, temperature variation of the solid and fluid phases within the packed bed with respect to time was considered. Figure 3 depicts this variation for the case with constant-temperature and constant-pressure inlet boundary conditions.

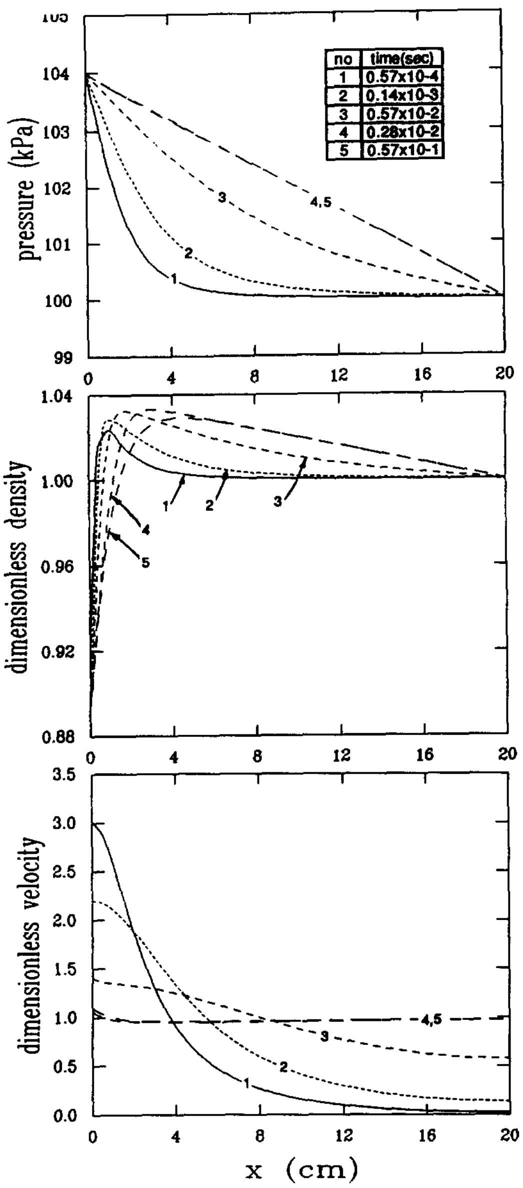

Figure 2 Variation of different field variables during *early stage*

For convenience, this case will be called Case I. Figure 4 shows the temperature variations for constant temperature and oscillating pressure at the inlet of the packed bed. This case will be called Case II. For Case II, the chosen parameters were $A=2$ kPa and $f=0.05$ Hz. For Case III, which is for constant-pressure but oscillating-temperature inlet boundary conditions, the temperature variations are shown in Figure 5. For this case we had $B=25$ K and $f=0.05$ Hz. Qualitatively, Figures 3 and 4 depict similar behavior for the solid- and fluid-phase temperature variations with larger temperature difference between the two phases at the beginning, and narrowing difference as the thermal front moves within the packed bed. This behavior is also valid for Case III in the downstream section of the packed bed but not at the entrance region because of the oscillating temperature inlet condition. At the entrance region, the temperature difference between the solid and fluid phases can increase or decrease due to the oscillating inlet fluid temperature and due to the fact that there will be a lag time for the response of the solid-phase temperature to this oscillation.

Figure 6 depicts the variation of the gas-phase density, pressure, and velocity at four different time levels for three different cases for Case II during one complete cycle of the pressure oscillation. These three cases involved different values for parameters A and f as shown in Figure 6. Cases in Figure 6a and 6b have the same amplitude but different frequency of inlet pressure oscillation, whereas cases in Figure 6b and 6c

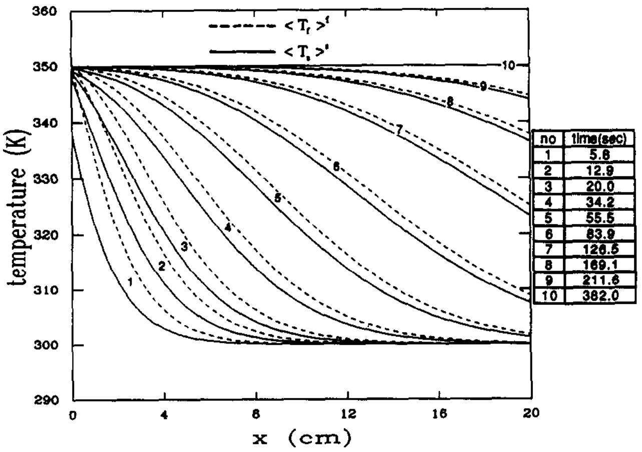

Figure 3 Temperature distribution in the packed bed, $A=0$, $B=0$

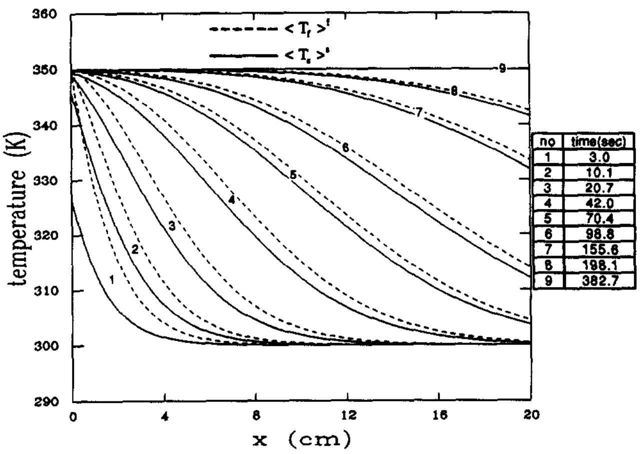

Figure 4 Temperature distribution in the packed bed, $A=2$ kPa, $f=0.05$ Hz

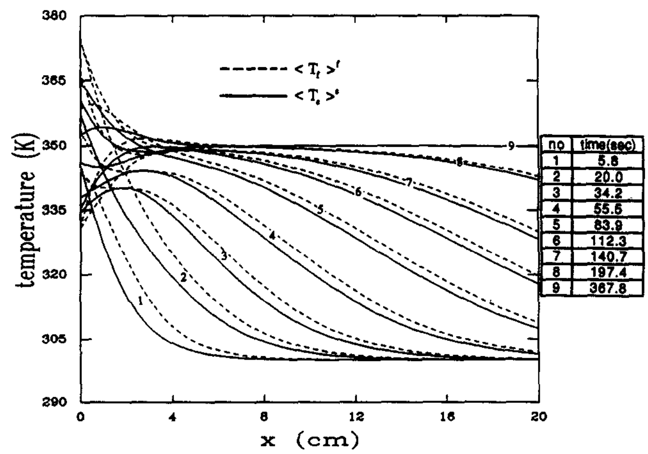

Figure 5 Temperature distribution in the packed bed, $B=25$ K, $f=0.05$ Hz

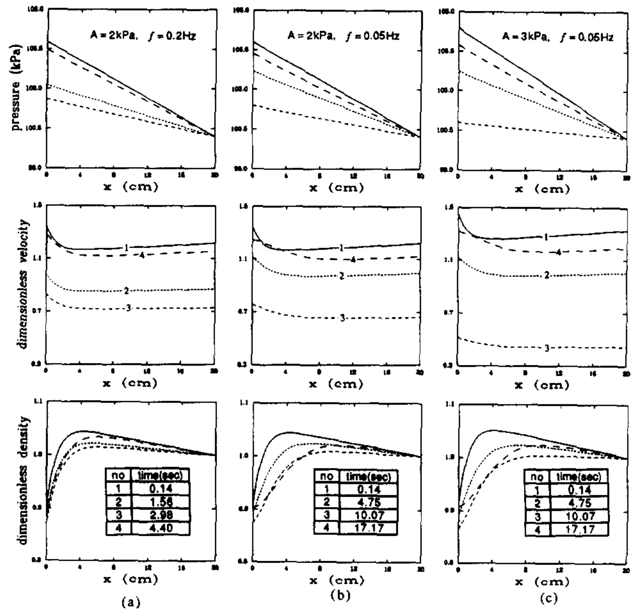

Figure 6 Variation of field variables during the first complete pressure cycle

have the same frequency but different amplitudes of inlet pressure oscillation. In each case the four time levels depicted were chosen so that they would span over one complete cycle of oscillation (the very first cycle in the process). Although the pressure distribution is nearly linear along the packed bed during the first three time levels, a careful examination of the figures at the fourth time level reveals that the pressure distribution in the packed bed picks up from the oscillating inlet condition and shows an oscillating behavior along the packed bed too, i.e., spatial oscillation in addition to temporal oscillation. Also, as can be expected, the range of variation of the gas-phase velocity is higher in the case with larger amplitude in the inlet pressure oscillation. The comparison of the range of velocity variation in Figure 6c with those in Figure 6a and 6b reveals this clearly. The variation in the gas-phase density can be explained by the use of the equation of state and the variation of the pressure and temperature along the packed bed. At the entrance region, the sharp decrease in the temperature at the beginning of the charging process requires an increase in the gas-phase density, since the rate of decrease in gas-phase pressure is less pronounced than that in temperature.

Figure 7 depicts the time history of the net energy storage per unit width of the packed bed for Case I and two cases of Case II. As expected, the asymptotic value of the total net energy stored in each case is the same. As the amplitude of the oscillations in the inlet pressure increases, the oscillations in the net energy stored become more pronounced, since the oscillations in effect result in oscillations in the mass flow rate of the gas phase through the packed bed. Larger frequencies, on the other hand, tend to smooth the variation in the net energy storage.

The variations in the rate of heat flow into and out of the packed bed for Case I and Case II with $A = 2$ kPa and $f = 0.05$ Hz are depicted in Figure 8. The oscillating behavior in this figure is that of Case II and the smooth variation is that of Case I. Although the heat flow rates into and out of the packed bed oscillate in Case II due to the oscillation in the inlet pressure and hence the mass flow rate, the average variation of each of these quantities is qualitatively very similar to those of Case I, i.e., the variation of the difference between the heat flowing into and out of the packed bed has the same trend in both cases.

The comparison of the rate of heat flow into and out of the packed bed for Case I and Case III is shown in Figure 9. For Case III in this figure, $B = 25$ K and $f = 0.05$ Hz. Again, due to the oscillating inlet temperature, the density and velocity of the gas and hence the mass flow rate into the packed bed oscillates. However, this oscillation in temperature has a less pronounced effect on the heat flow rates into and out of the packed bed than the oscillation in the inlet pressure has, as shown in Figure 8. The difference between the scales of Figures 8 and 9 should be noted. Part of the reason for this behavior may be attributed

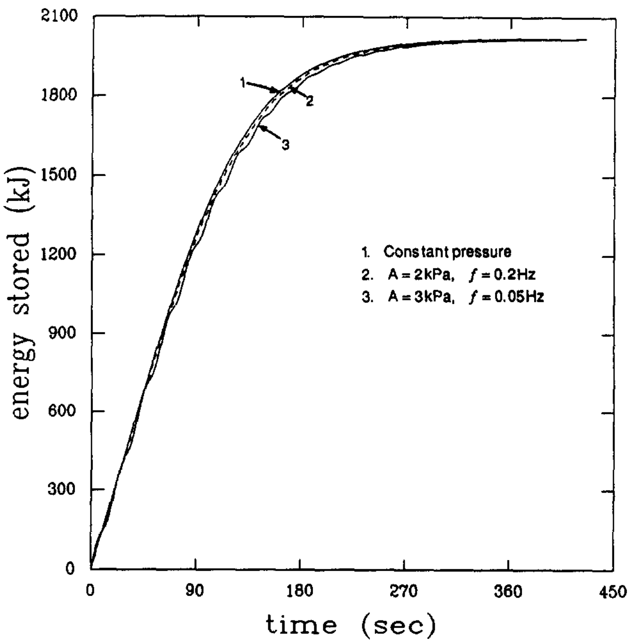

Figure 7 Time history of energy storage in the packed bed

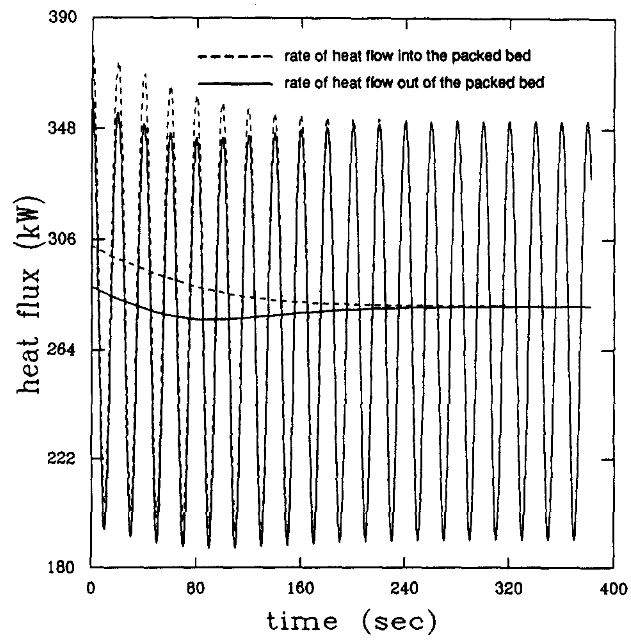

Figure 8 Time history of heat flow rates into and out of the packed bed

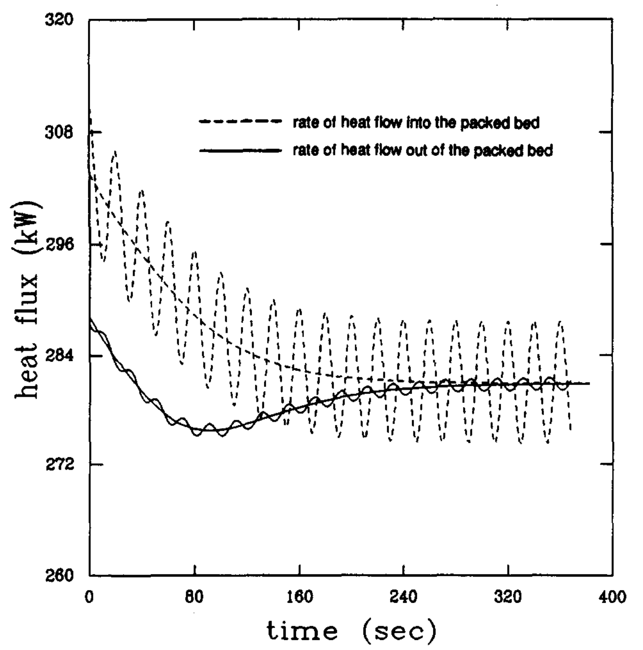

Figure 9 Time history of heat flow rates into and out of the packed bed

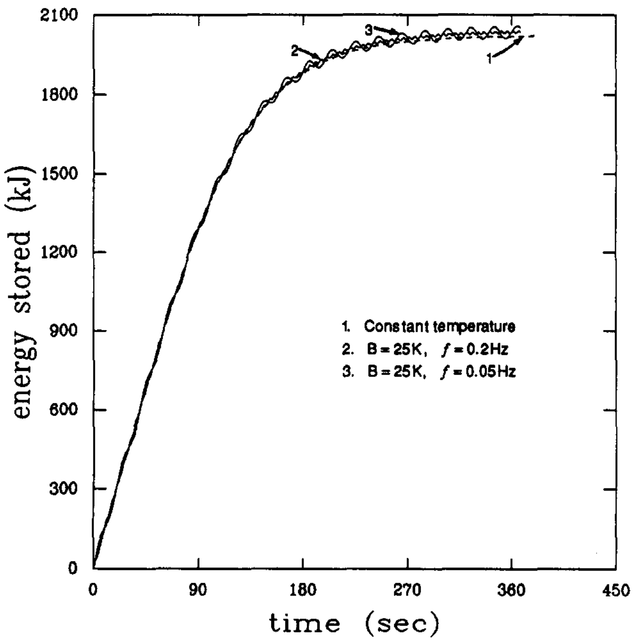

Figure 10 Time history of energy storage in the packed bed

to the fact that, in calculating the gas-phase velocity, the pressure gradient term in the gas-phase momentum equation is much more dominant than the inertia term, which involves the gas-phase density, which in turn varies in proportion to the temperature. Thus the former affects the variation in the gas-phase velocity more, causing a more pronounced oscillation in the mass flow rate of the gas phase.

In Case III we cannot talk about the complete thermal charging of the packed bed due to the oscillating inlet-temperature boundary condition. Because at the entrance region of the packed bed, due to this oscillation, the solid temperature also oscillates (not necessarily at the same frequency and amplitude, because the heat capacity of the solid phase is much larger than that of the working fluid, and therefore the temperature of the solid phase cannot follow the temperature of the incoming gas at the same frequency and amplitude). Yet, one can speak of a pseudocharging of the packed bed. Figure 10 depicts this pseudocharging behavior (representing the net energy stored within the packed bed). This behavior is different from that of Case II, where although the gas-phase inlet pressure is oscillating, the gas-phase inlet temperature is kept constant at the highest value that can be attained by the packed-bed particles; consequently, there is continuous thermal energy storage within the packed bed until the packed bed is thermally fully charged. However, in Case III, due to the variation of the solid temperature in the entrance region, there is alternating energy storage and removal from the packed bed.

In order to find out whether the qualitative behavior of the energy storage characteristics changes with particle Reynolds number (Re_p), cases with different nominal Re_p were investigated. Higher-particle Reynolds numbers were obtained by increasing the mean inlet pressure. The results are depicted in Figure 11 for three different cases in which the amplitude of the inlet pressure oscillations was the same. As can be seen, the qualitative behavior is similar in each case, although higher nominal Reynolds numbers, meaning higher mass flow rates, cause faster charging of the packed bed. The effect of the pressure oscillations is seen most clearly in the case with lowest Re_p, since the amplitude of the oscillations is largest relative to the global pressure difference applied across the packed bed for that case.

Conclusions

The dynamic response of sensible heat storage packed beds with oscillating inlet boundary conditions has been studied numerically for forced convective flow of a compressible fluid. A finite-difference scheme with uniform grid size was employed. It was found that the average energy storage behavior did not have major differences in the cases of constant or oscillating inlet boundary conditions, although the field variables showed

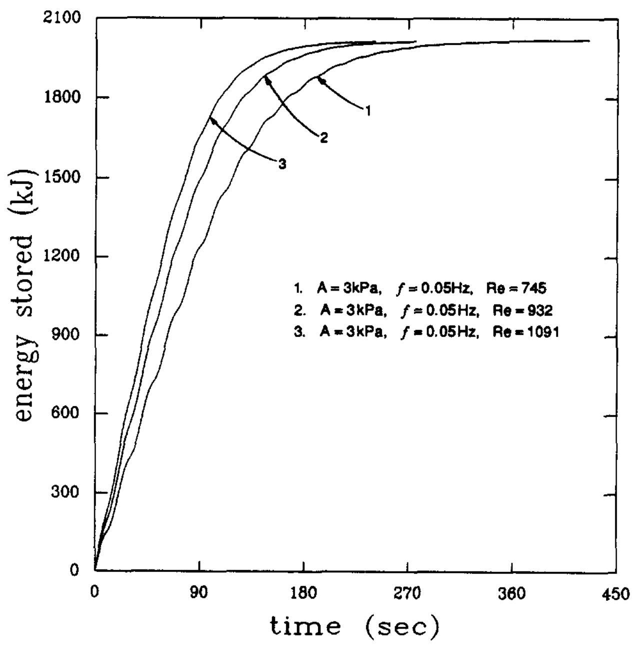

Figure 11 Time history of energy storage in the packed bed

oscillating behavior in cases of oscillating boundary conditions. As expected, the variation of the energy storage was found to become smoother as the amplitude of the oscillation of the inlet condition decreased and/or as the frequency increased. It was also observed that, due to the nature of the governing equations, the response of the field variables was more sensitive to the amplitude of the fluid pressure variation than to that of the fluid temperature variation at the inlet of the packed bed.

Acknowledgment

The support by Aero Propulsion Laboratory of the U.S. Air Force under contract No. F33615–89–C–2949 and the grant provided by the Ohio Supercomputer Center for the numerical computations for this investigation is acknowledged and appreciated. The authors also acknowledge Dr. W. Chang and Dr. J. E. Beam for their support and help with this work.

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